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Section 3
ELECTRICAL ENGINEERING, ELECTRONICS AND AUTOMATION

SYNTHESIS OF THE SERVO SYSTEM WITH THE WAVE CHANNELS

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Abstract

It is shown that, using results of the measurements "littered" with noise of linear combinations of variables of a condition of object, their estimates can be received, applying the filter consisting of "model" of initial system and a signal of feedback, proportional to a difference between the valid measurement and its assessment. Thus the law of optimum control is in result of minimization of the square functionality of quality of system average on ensemble. The system possessing that property is synthesized that in the absence of other indignations and noise of supervision continuous indignation is always compensated in such a way that the error of regulation or tracking is equal in the established state to zero that is reached due to integrating operation of the proportional and integrated and differential regulator. The method of calculation of management in watching system with wave channels, proceeding from the assumption is offered that the full vector of a state is known (it is restored on the basis of the known theorem of a separation), and also proceeding from replacement of the valid condition of system with the restored.

Keywords: operating influences; watching system; wave channels; noise-alarm situation,

INTRODUCTION

As it is known [1÷6], for optimum control of dynamic system it is necessary to know its state. However in practice often separate variable conditions of OC or can't be measured directly, or are measured with a big error. Usually results of measurements which can be executed represent functions of variables of a state and contain random errors. As a rule, the system is also subject to impact of casual indignations.

ESTIMATION COMBINATIONS OF VARIABLES OF OBJECT CONDITION

Thus, for ensuring optimum control it is necessary to estimate variable states either on too small, or on too large number of measurements which are inexact and represent functions of variables of a state. Using results of the measurements littered with noise of linear combinations of variables of a state, the assessment of their state can be received, applying the filter consisting of model of initial system and a signal of feedback, proportional to a difference between the valid value of result of measurement and its assessment [7].

The combination of the optimum filter and the optimum determined regulator represents the regulator with feedback, optimum that is an average on ensemble for a linear task with square functionality and additive Gaussian white noise. In this case the principle of stochastic equivalence, or the theorem of divisibility [8] is fair.

The law of optimum control is in result of minimization of the square functionality average on ensemble:

$$J = E\left[\frac{1}{2}\lim_{T\to\infty}\frac{1}{T}\int_t^{t+T}\left(x^T Qx + u^T Lu\right)dt\right], \qquad (1)$$

where E – a symbol of a population mean.

The population mean of value J is less exact measure of quality of system. Nevertheless, it is a convenient measure which can serve as the good compromise solution of a question.

At an periodicity assumption the functionality from the equation (1) will correspond as:

$$J = \frac{1}{2}E\left[E\left(x^T Qx + u^T Lu\right)\right] \qquad (2)$$

or

$$J = \frac{1}{2}S_P\left[QX + G^T LGX\right], \qquad (3)$$

where x – a deviation of a condition of process from desirable; S_P – a matrix trace, i.e. the sum of the elements of

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its main diagonal; X – a stationary covariance matrix of variables of a state; G – required matrix of optimum feedback.

Optimal control strategy in this case is

$$U^* = -L^{-1}\widetilde{B}^T Px = Gx\,. \quad (4)$$

Here: U^* – optimum control; P – coefficients which find integration in the return time of the equations of Rikkaty:

$$\dot{P} = -P\widetilde{A} - \widetilde{A}^T P - Q + P\widetilde{B}L^{-1}\widetilde{B}^T P\,. \quad (5)$$

The result of minimization of functionality is well-known from literature [9] and includes management: based on the dynamics of the vector perturbation, or its evaluation, and feedback control based on the state variables or estimates.

We will note that the decision for a matrix of G doesn't depend on that, the system as casual or as determined with the entry conditions defined or by means of a covariance matrix for casual system, or values of all variables of a state in an initial timepoint for the determined system [10] is considered.

In a general view the set part of the watching system with wave channels (WSWC) is described:
state equation

$$\dot{x} = f(x, U, V, t) \quad (6)$$

supervision equation

$$y = g(x,t) + w\,, \quad (7)$$

where $x = [x_1 \vdots x_2 \vdots x_3]^T - (n_1 + n_2 + n_3)$-a measured vector of the state which components are: x_1–n_1-a measured vector of a condition of operated part; x_2–n_2-a measured vector of indignation (surrounding conditions); x_3–n_0-a measured vector of a reference signal (uncontrollable part of system) – the purpose (fig. 1).

The rational structure of a vector of a state is defined with requirements to WSWC and information on results of supervision. So, angular provisions, speeds and accelerations of mechanical, electric and optical axes of the antenna (an equal-signal zone - ESZ) can be vector elements x_1; frequency or phase and their derivatives. Reference values of coordinates and their derivatives, corresponding to a vector x_1 will be vector elements x_3 in this case. Behind a vector x_3 tracking is conducted. The vector x_1 characterizes behavior of environment – the indignations operating on set part of WSWC. Elements of a vector of y are the measured components dependent from x_1 and x_3. The vector x_1 characterizes operated part of system, and x_3 – uncontrollable – the purpose [10]. The vector of w reflects noise of measurements. At measurement of a reference signal it is noise of intakes, angular and amplitude noise of the purpose, a dying down of signals and other radio noise. The vector of management U is formed by the regulator providing optimization of set criterion of quality of functioning of system.

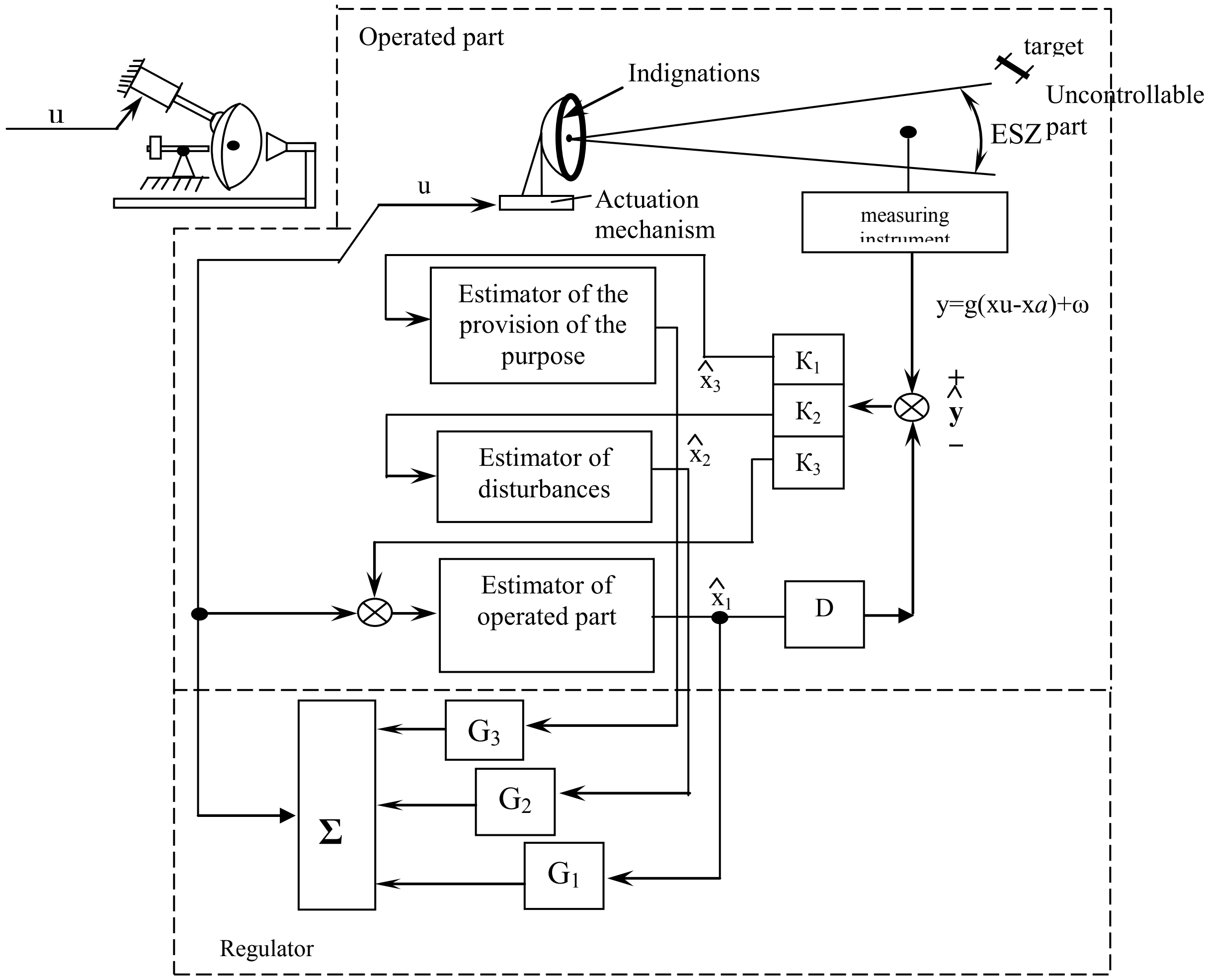

Fig. 1. Scheme of watching system with wave channels

Vector functions *f(·)* and *g(·)* generally nonlinear, changing in time. Exact look and values of parameters of the above-stated equations depend on purpose of WSWC.

The watching system works with wave channels (fig. 1) as follows:

the estimator gives $\hat{x} = \left[\hat{x}_1 \vdots \hat{x}_2 \vdots \hat{x}_3\right]^T$ out an assessment of the valid condition of the set part of system *x*, received on the basis of all measurements of *y* given by the time of estimation subject to hindrances. This assessment is used then in the regulator at calculation of a signal of the management *U* influencing operated part of system so that the accepted criterion was minimized. It is supposed that the equations of an estimator and the regulator can be received independently. The similar assumption can lead to the decision which isn't strictly optimum since WSWC are nonlinear systems. However it allows to receive numerical the realized algorithm leading to quite acceptable results.

It is supposed also that it is possible to measure only some combinations a component of a condition of the set part of WSWC which besides are littered with additive noise. Information about x_3 contains in WSWC, as a rule, in narrow-band (generally – casual) a signal. At the exit of an intake we have:

$$y_3 = g_3(x_3, w_3, t)\,. \qquad (8)$$

For receiving an assessment $\hat{x}_3$ a signal x_3 optimum measuring instruments (discriminators) with the device of an assessment [3] were widely adopted now.

Let the linearization of WSWC model be described by the equations:

$$\dot{X}_1 = Ax_1 + BU + CV\,, \qquad (9)$$

$$y_1 = D_1 x_1 + w_1\,, \qquad (10)$$

where x_1-*n*-a measured vector of a state; *U*-*r*-a measured vector of management; *V*-*s*-a measured vector of indignations; y_1-m_1-the measured vector of measurements connected with x_1; w_1-m_1-a measured vector of noise of measurement.

The surrounding conditions creating indignations are described by means of the following equations:

$$\dot{x}_2 = Ex_2 + Fn\,, \qquad (11)$$

$$V = Hx_2\,, \qquad (12)$$

$$y_2 = D_2 x_2 + w_2\,, \qquad (13)$$

where (x_2–*g*)-a measured vector of indignations; *n(t)*-a vector of white noise with zero average value; (y_2–m_2)-a measured vector of the measurements connected with x_2; (w_2–m_2)-a measured vector of noise of measurement.

The behavior of the purpose – a vector x_3 is similarly simulated. In the equations (9)÷(13) of *A*, *B*, *C*, D_1, D_2, *E*, *F* and *H* value represent the matrixes which dimensions are defined by dimensions of the corresponding vectors. Ratios (11)÷(13) are model of casual indignations of functions, other than white noise. The output signal of stationary Gaussian Markov process of the first order has exponential correlation function. It specifies a way of research of tasks in which noise on an entrance is casual, but not white. Using forming filters of the first or highest order (stationary or non-stationary) with white noise on an entrance, we receive as output signals correlated in time or the painted noise, i.e. Gaussian Markov process.

Almost any correlation function of noise for practical purposes rather well can be approximated the corresponding choice of coefficients in the forming filter. The condition of all system will be defined in this case by an expanded vector of condition $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ or (with the purpose) $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$. The expanded vector of a state is also Gaussian Markov process. The condition of system with expanded space of the states, including both object (process), and external influence, is described in this case by means of the following equations:

$$x = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dfrac{A}{O} & \Big| & \dfrac{CH}{E} \end{bmatrix} x + \begin{bmatrix} \dfrac{B}{O} & \Big| & \dfrac{O}{O} \end{bmatrix} \begin{bmatrix} U \\ O \end{bmatrix} + \begin{bmatrix} \dfrac{O}{F} & \Big| & \dfrac{O}{O} \end{bmatrix} \begin{bmatrix} n \\ o \end{bmatrix}, \qquad (14)$$

$$y = \begin{bmatrix} \dfrac{D_1}{O} & \Big| & \dfrac{O}{D_2} \end{bmatrix}, \qquad (15)$$

$$w = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix};\; y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}. \qquad (16)$$

The equations (10), (11) can be copied in a look:

$$\dot{X} = \tilde{A}x + \tilde{B}U + \tilde{F}n\,,\; y = \tilde{D}x + w\,. \qquad (17)$$

Thus process *x* and a hindrance of *w* rely the independent.

The optimum linear equation with square functionality for such system has a form:

$$u^* = Gx = \left[G_1 \vdots G_2\right] \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}. \qquad (18)$$

The equations defining structure and characteristics of the optimum filter can be received in various ways. As their conclusion is rather bulky, we will be limited only to discussion of the end results. The structure of the optimum filter which is carrying out an assessment on a method of the smallest squares is described by the vector differential equation:

$$\dot{\hat{x}} = \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \tilde{A}\hat{X} + \tilde{B}U + \tilde{K}(y - \tilde{y})\,,\; \hat{X}(t_0) = 0\,, \qquad (19)$$

where $\hat{X}$ – an assessment $\hat{x}$; $\tilde{K}$ – as appropriate calculated matrix.

The difference $(y - \hat{y})$ can be interpreted physically as a difference between observed and predicted values *y*, received at restoration of variables of a state *x*. At the expense of the corresponding choice *K* the error recovery $(x - \hat{x})$ can be made zero for any conditions of model of the equation (19). Initial value of a vector $\hat{X}(t_0)$ in the equation (19) usually is necessary zero. In the equation (19) matrix $\tilde{K}$ is defined from a ratio:

$$\widetilde{K}(t)=\begin{bmatrix}K_1\\K_2\end{bmatrix}=\Delta X(t)\widetilde{D}^T w, \qquad (20)$$

where dimension of K_1 – $n\times m$, and K_2 – $g\times m$, ΔX and $w\delta(\tau)=E\left[w(t)w^T(t+\tau)\right]$ – square covariance matrixes of errors of an assessment of a state and measurement noise respectively.

The matrix of proportionality $\widetilde{K}$ actually characterizes a ratio between uncertainty of a state ΔX and uncertainty in w measurements.

The matrix of covariance $\Delta X(t)$ is defined as:

$$\Delta X(t)=E\left[\Delta X(t)\Delta X^T(t)\right]= \\ =E\left\{\left[\hat{X}(t)-X(t)\right]\left[\hat{X}(t)-X(t)\right]^T\right\}= \qquad (21) \\ =E\begin{bmatrix}(\hat{x}1-x_1)^2....(\hat{x}_1-x_1)(\hat{x}n-x_n)\\ ....\\ (\hat{x}_n-x_n)(\hat{x}1-x_1)...(\hat{x}_n-x_n)^2\end{bmatrix}.$$

Here it is supposed that the operator of a population mean is applied to each element of a matrix. Diagonal elements of a matrix are dispersions a vector component. The elements standing out of a diagonal are the mixed elements of the second order. Matrix $\Delta X(t)$ – symmetric, various elements consisting of $n(n+1)/2$. Thus, elements of this matrix are dispersions and mutual dispersions of errors of a filtration of expected components of a vector of $X(t)$. Values of elements of a matrix $\Delta X(t)$ can be determined as a result of the solution of the nonlinear equation like Rikkaties matrix equation:

$$\Delta X(t)=\widetilde{A}\Delta X(t)+\Delta X(t)\widetilde{A}^T+\widetilde{F}N\widetilde{F}^T-\Delta X(t)\widetilde{D}^T w^1\widetilde{D}\Delta X(t)$$

$$\Delta X(t_0)=\Delta X_0, \qquad (22)$$

where $N\delta(\tau)=E\left[n(t)n^T(t+\tau)\right]$.

The equation (22) characterizes change of errors of a filtration in time. The first two composed right members of equation (22) represent change of a covariance matrix of the errors recovery, connected with dynamics of system. The third composed is a change of a covariance matrix of the errors recovery, caused by indignation of V, operating on system. And, at last, the last composed characterizes change of a covariance matrix of errors recovery as a result of carried-out measurements. It depends on a choice of matrix coefficient of strengthening $\widetilde{K}$.

For the solution of the equation (22) it is necessary to set initial value $\Delta X(t_0)$ of a matrix of dispersions of mistakes. If at the time of $t=t_0$ process $\hat{X}(t_0)$ at the exit of the filter is equal to zero, that, apparently from the equation (21), the matrix $\Delta X(t_0)$ is equal to a matrix of dispersions a component of filtered process $X(t)$ at the time of $t=t_0$, i.e.

$$\Delta X(t_0)=E\left\{\left[-X(t_0)\right]\left[-X^T(t_0)\right]\right\}. \qquad (23)$$

The structure of the full system consisting directly of operated process, surrounding environment, estimator of states (filter of Kalman-Bucy) and optimum feedback, is shown in figure 2. It is brought for a case when the state x_2 environment isn't measured.

Thus, for definition of management it is necessary to solve two ordinary differential equations: the equation (22) for a matrix ΔX under the set entry condition and the equation (4) for a matrix P under the set final condition. Here coefficients of $P(t)$ find integration in the return time of the equations of Rikkaty (5).

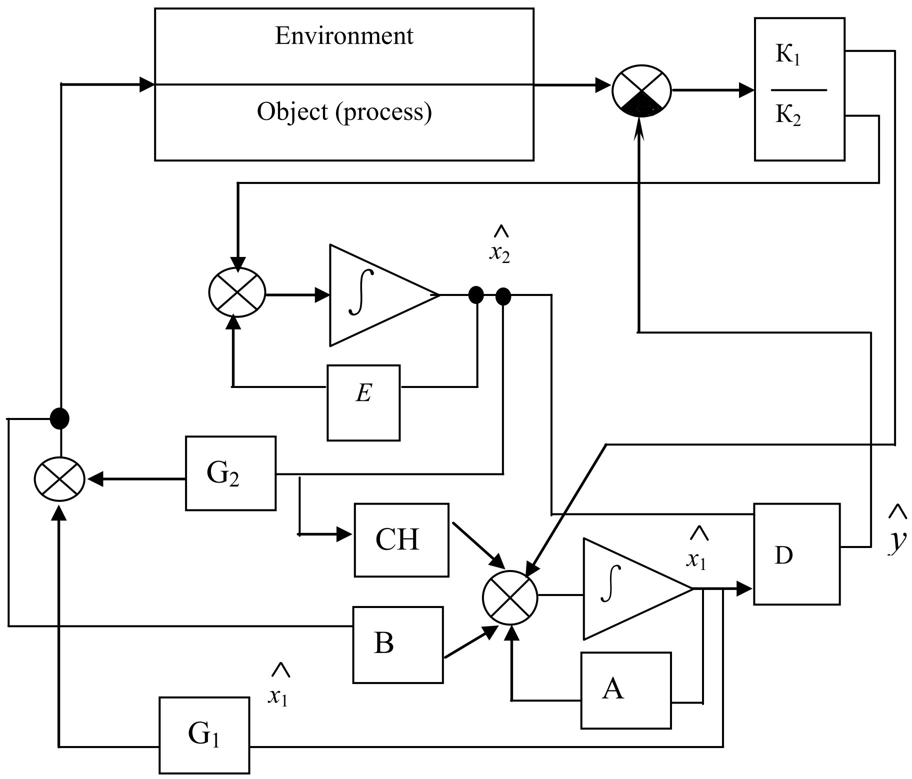

Fig. 2. Structure of the control system

The key to practical calculation of the filter of Kalman consists in a right choice of sizes of elements the covariance of matrixes of *w* and *N*. As a rule, width of a pass-band of the filter quantitatively depends on the relation $\|w\|\|N\|^{-1}$ [9]. The it is more, the less width of a pass-band of the filter. The compromise depends on this ratio between the speed of restoration of a state and resistance to noise of supervision. Shift of poles of supervision in any half of the complex plane leads to increase in a pass-band and, therefore, restoration speed.

In practice of a matrix of *N* and *w* choose so that poles of an estimator and the regulator were from the beginning of coordinates at distance of about one about [11]. It is uneconomical to have regulation with very high speed when process of restoration is slow and vice versa. In particular, when noise of supervision much more noise of an exciting state, poles of supervision are rather close to the beginning of coordinates, and process of restoration is slow. If now to achieve speed of the regulator a little bigger, than from the observer, it is necessary to expect that the regulator will restrain the observer. Further increase of speed of the regulator will increase only a mean square of an entrance variable without change of a mean square of an error of regulation. On the other hand, when noise of supervision is small, the admissible mean square of an entrance variable becomes limiting factor in system.

Thereby speed of the regulator is limited and there is inexpedient a choice of the observer with very big speed even if characteristics of noise allow it.

In some cases it is physically possible to assume that in the equations (2) ÷ (4) E = 0; F = 1; H = 1; i.e. $\dot{x} = In; \vartheta = x_2$,
where I – a single matrix.

It corresponds to a case of continuous indignations. Often these indignations are caused by inaccuracy of determination of the corresponding nominal sizes of an entrance variable, a state and operating variable. The resultant established optimum observer is described by the equations:

$$\dot{\hat{x}}_1(t) = A\hat{x}_1(t) + Bu(t) + \hat{x}_2(t) + k_1\left[y(t) - D_1\hat{x}_1(t)\right];$$

$$\dot{\hat{x}}_1(t) = k_2\left[y(t) - D_1\hat{x}_1(t)\right]. \qquad (24)$$

The system represented in fig. 2, in this case will have structure in proportion – integrated-differential management on many variables. The system possesses that property that in the absence of other indignations and noise of supervision continuous indignation is always compensated so that the error of regulation or tracking was equal in the established state to zero. It is reached for the account of "integrating action" regulator.

Matrixes of K_1 and K_2 are calculated directly from Rikkaty's equations (16), (21). As a rule, it demands use of the COMPUTER. The high-speed differential feedback providing a stable condition of full system, is defined by devices of an assessment of variables of a condition of process which don't depend on mistakes when modeling system.

CONCLUSIONS

In *summary* it should be noted that unlike the optimum regulator the optimum observer can be realized in RMV as the equation, (16), (21) are the differential equation with the set entry conditions whereas in a problem of optimum regulation it is necessary to solve Rikkaties equation under the set final conditions in the return time.

REFERENCES

[1]. Malkin I.G. Teorija ustojchivosti dvizhenija [The theory of motion stability]. M.: «Editorial» [Moscow: "Editorial"], 2010. -432 p.

[2]. Kalman R.E. Theory of control Systems, IFAC, London, Butter worth and co., Ltd; 1960.

[3]. Jusupbekov A.N., Ataullaev A.O. Intellektual'naja sledjashhaja sistema s volnovymi kanalami [Intelligent tracking system with wave channels], Tashkent: Monografija, TashGTU [Tashkent: Monograph, Tashkent State Technical University], 2015. -152 p.

[4]. Taylar F. J., Pries T. H., Sluang C. H. Optimal Wind Velocity Estimation, Automatica, vol, 1970.

[5]. Balchen J. G. Regulating technical, Bind 1, Tapir, Trondheim, 1971.

[6]. Puzyrev V.A. Cifrovye samonastraivajushhiesja sistemy [Digital self-adjusting system]. M.: Izdatel'stvo «Radio i svjaz'» [Moscow: Publishing House "Radio and Communication"], 1992.

[7]. Kuz'min S.Z. Cifrovaja radiolokacija. Vvedenie v teoriju [Digital radar. Introduction to theory], Kiev: Izdatel'stvo VC [Kiev: Publishing VTS], 2000.

[8]. Kazarinov Ju.M., Sokolov A.I., Jurchenko Ju.S. Proektirovanie ustrojstv fil'tracii radiosignalov [Device filtering signals]. L.: Izdatel'stvo Leningr. un-ta [Leningrad: Publishing of Leningrad’s University], 1985. -160 p.

[9]. Perov A.I. Statisticheskaja teorija radiotehnicheskih sistem [Statistical theory of radio systems]. M.: Radiotehnika [Moscow: Radio Engineering], 2003. - 398 p.

[10]. Guljamov Sh.M., Jusupbekov A.N., Rashidov Ju.R., Ataullaev A.O. Metodika rascheta upravlenij v sledjashhej sisteme s volnovymi kanalami s uchetom slozhnyh pomeho-signal'nyh uslovij [Calculation of Control Actions in a Servosystem with Wave Channels Subject to Complicated Disturbance Conditions]. Zhurnal «Promyshlennye ASU i kontrollery» [The journal "Industrial Automatic Control Systems and Controllers»], 2014. -pp. 24÷30

[11]. Tihonov V.I. Optimal'nyj priem signalov [Optimal signal reception]. M.: Izdatelstvo: «Sovetskoe radio» [Moscow: Publishing House «Soviet Radio»] 1983. - 320 p.