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# INFLUENCE OF INERTIAL FORCES ON FLOW OF ELECTRICALLY CONDUCTIVE LIQUIDS IN A TRANSVERSE MAGNETIC FIELD

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### ARTICLE INFO

## ABSTRACT

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In a whole range of hydromechanical equipment, there are working sections in which the flow of an electrically conductive fluid in a transverse magnetic field is realized over short sections of the channel. This kind of flow differs in a number of features from a stabilized flow. These peculiarities are due to the fact that, unlike a stabilized flow, fluid motion is implemented in the field of action of inertia forces and forces of a magnetic nature, that is, ponderomotive forces. Depending on the rheological properties of the fluid and the magnitude of this type of force, a hydrodynamic initial section is formed, which has peculiarities of the distribution of velocities, stresses and pressures. In this case, the length of this section plays an important role, since depending on it staying time of the liquid in this section can be determined, which is important in a number of technological processes associated with metallurgy, biomechanics and mechanotronic systems. In this regard, the task of forming ideas about the processes of magnetic hydrodynamics for this case is relevant. An attempt to study a similar problem in this work is devoted to the creation of both a mathematical and physical model of this flow. Based on experimental data, taking into account the rheological properties of working fluids, an attempt was made to determine the criteria characterizing this process and their influence on the hydrodynamic initial section.

**1. INTRODUCTION** 

One of the most important problems in hydrodynamics of the flow of viscous and abnormally viscous liquids is problem of hydrodynamic initial section. A feature of flow in this case is a factor associated with manifestation, in addition to the forces of viscous friction, inertia forces from convective acceleration. As known, presence of inertia forces leads to destabilization of the flow, which manifests itself in deformations of the field of velocities and stress fields. A number of studies have been devoted to this problem [3], [4], [5], [6], [7], [8], [9]. At the same time, similar studies, in which, in addition to the forces of viscous friction and inertia forces, there are also forces of an electromagnetic nature, known as ponderomotive forces, unfortunately, are not enough. It should be noted that relevance of problems associated with influence of ponderomotive forces is very high and is associated with design of different technological equipment and in medicine, when blood is considered as an electrically conductive liquid [10], [11]. In this regard, the problem

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arises of assessing the degree of influence on the flow, in addition to the forces of viscous friction, inertia forces and ponderomotive forces. Depending on the ratio between these quantities, the nature of the flow can be different. In this regard, the purpose of this research is to study the degree of manifestation of the presented forces on the flow. Considering the above, an attempt was made to assess the impact of the presented forces on the destabilization of the flow in the area of the hydrodynamic initial section. The concept of the initial section is based on the data presented in [1], [2], [4], [7], [8], [9].

### 2. EXPOSITION

One of the most important points of view is a theory based on the development of a boundary layer in the initial section, the thickness of which varies from zero at the inlet to a value corresponding to half the thickness of the flow cross section, as it shown on the Figure 1.

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Fig. 1. Qualitative picture of the velocity distribution according to Targ data [6]

Within the boundaries of the boundary layer, the flow velocity is decelerated due to viscous friction forces; at the same time, the flow core is moving at an accelerated rate. With an increase in the thickness of the boundary layer, the dimensions of the flow core decrease. Thus, when describing the flow in the hydrodynamic initial section, it becomes necessary to assess the degree of influence of inertia forces from convective acceleration on the manifestation of viscous friction forces. Research by a number of authors has shown that for electrically conductive liquids, the influence of a magnetic field leads to a braking effect. Thus, depending on the magnitude of the magnetic field and inertia forces, it should be expected that the deformation of the velocity diagram differs from the traditional representation in the initial section shown in Figure 1. Such an estimate makes it possible to calculate the length of the hydrodynamic initial section as a function of the geometric dimensions of the channel and the Reynolds number. Table 1 presents expressions characterizing the length of the initial section in the absence of ponderomotive forces.

In the presence of a transverse magnetic field and the condition of the electrical conductivity of the liquid, an additional force is added to these two components, associated with the intensity of the transverse magnetic field.

Table 1 (	Calculation	formulas fo	r the	initial section	
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= = = =			
Authors	Initial section length $(L_{IS} = const \cdot Re \cdot D)$ <b>const</b> values		
Schiller	0,0575		
Bussinesque	0,12		
Campbell, Slattery	0,061		
Nikuradze	0,0625		
Targ	0,08		
Goldstein	0,0575		
Langhaar	0,0575		

Despite the difference in the nature of the inertia forces from convective acceleration and magnetic forces, these two components associated with the transverse magnetic field can significantly affect the nature of the flow in the hydrodynamic initial section. Depending on the ratio between the forces of inertia and forces with an electromagnetic nature, the influence of the magnetic field can lead to the manifestation of both deceleration of the flow, and leading to its acceleration. For the hydrodynamic initial section in the considered case, the equations of motion can be represented in the following form:

$$\begin{cases} \rho(u\nabla)u = -\nabla p + \mu\Delta u + \frac{1}{c}[j \times B], & div \, u = 0\\ j = \sigma\left(-\nabla \varphi + \frac{1}{c}[u \times B]\right), & div \, j = 0 \end{cases}$$
(1)

$$rot B = \frac{4\pi}{c} j, \qquad div B = 0 \qquad (2)$$

where  $u = u_0(x, y)$  - velocity distribution, *B* - magnetic field induction, *j* - current density,  $\varphi$  - potential.

In the Cartesian coordinate system, this equation can be written as follows

$$\begin{cases} \rho \left( u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} \right) = -\frac{dp}{dx} + \mu \frac{\partial^2 u_x}{\partial y^2} - \frac{\sigma B_0^2}{c^2} \cdot \frac{\partial u_x}{\partial y} \\ \frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} = 0 \end{cases}$$
(3)

where the mass forces associated with the manifestation of the magnetic field are represented by the term  $\rho(\vec{u}\nabla)\vec{u}$ ;

$$\frac{1}{c} \left[ \vec{j} \times \vec{B} \right].$$

The solution of this equation for the case when there are no inertia forces from convective acceleration was obtained in [7] and has the following form.

The analysis of the flow in the laminar regime in the initial section is based on assumption that the flow parameters in a transverse magnetic field and the flow direction do not change fast enough. This makes it possible to study the flow in each cross section on the basis of a twodimensional theory, in which the velocity is assumed and the induced magnetic field is directed predominantly along the channel axis.

The deformation of the velocity diagram essentially depends on the Hartmann number. At low Hartmann numbers, the magnetic field has little effect on the length of the initial section, that is, at low Hartmann numbers, the calculation in the initial section can be carried out according to the formulas in the absence of a magnetic field. The degree of influence of the magnetic field was quite thoroughly studied in [4]. Taking into account the peculiarities of the influence of the magnetic field on destabilization, the length of the initial section can be determined by the dependence, where it is a function of not only the Reynolds number, but also the Hartmann number:

$$L \approx Re \frac{a}{Ha} \,. \tag{4}$$

As can be seen from the presented figure 3, with an increase in the Hartmann number under the influence of a magnetic field, a quasi-solid flow zone appears, which is characteristic of a viscous-plastic fluid. Using expressions characterizing the friction force, a law was obtained that characterizes the dependence of the friction coefficient on the Hartmann number:

$$c_f = \frac{2F_{fr}}{\rho u^2}.$$
(5)

Thus, it is possible to draw a conclusion about the influence of the magnetic field on the friction forces in the flow of an electromagnetic fluid. Finally, it should be noted that analyzing the currents under consideration, the authors of the work made the following conclusions:

• pressure forces in the quasi-solid zone are balanced by mass force  $[\vec{j} \times \vec{B}]$ ;

• at the boundary of the quasi-solid zone, the tangential stress equals zero;

• as the Hartmann number increases, the quasi-solid flow zone expands, and thus the velocity profile becomes flatter as it increases.



Fig. 2. Dependency graph  $u_x = f(x)$ 



#### **3. CONCLUSION**

Based on carried out researches, was obtained an idea about the characteristics of the flow of viscous and anomalously viscous liquids in the hydrodynamic initial section in the field of action of ponderomotive forces. Based on the data of distribution of the velocity field, were drawn conclusions about the length of the hydrodynamic initial section as a magnitude depending on the Reynolds and Hartmann criteria. It is shown that with increasing of Hartmann number under the influence of a magnetic field, a zone of quasi-solid flow appears, which is characteristic for a viscous-plastic fluid. Considering these features, dependence (5) can be recommended for determining the friction coefficient.

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