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COMPENSATION OF DELAY IN MULTIVARIABLE CONTROL SYSTEMS USING THE METHOD OF DYNAMIC GRAPHS

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ABSTRACT

In this paper, we study the features and possibilities of dynamic graph models application to solve the problem of designing multivariable systems with compensation for a time delay influence. As one of the most important varieties of delayed systems, we consider the water object control systems. The use of classical methods for calculating controllers for such systems leads to significant difficulties and requires cumbersome transformations. Developed in this paper, the method of dynamic graph models is based on the premise that such systems should be considered from the point of view of discretization not only of signals but also of the system structure. This approach allows to synthesize the control laws according to the chosen optimality criteria and to take into account the characteristics and properties of real control objects as closely as possible.

INTRODUCTION

Controlled processes of irrigation and hydro-power systems are described, generally, by partial differential equations that take into account such basic features of distributed processes as wave transfer of water flow, change of pure delay along canals, the influence of reflected waves on the dynamics of processes and so on [1–3]. The exact solutions of the equations have usually a complicated cumbersome kind as well as their application for analysis and synthesis of control systems comes across the considerable difficulties [4].

That is why, in practice, is applied the approximation of transfer functions of the complex systems with the distributed parameters with the aid of lumped parameters systems transfer functions and equivalent constants of the pure time delay. The presence of delay, as it is well known, worsens the stability and the process dynamic properties [5–14]. In consequence one of the first priority tasks, arising, when water objects control systems designing, shall be the time delay negative impact compensation task.

The other essential requirement comes as the transients time decrease, within the system or the task of maximum system response speed. The system parameters and control actions should be chosen so that the transient processes have aperiodic character, since overshooting may lead to idle discharges or shortage of water.

An important factor is also the choice of the functioning law of the controllers. At small values of equivalent pure delay, controllers with a continuous control law can be used. The latter is the unacceptable for irrigation objects with the allocated to considerable distances sensors. In this regard, widely use systems with discrete operating mode [15–18]. In the article we consider the calculation technique of the specified systems digital controllers with the aid of dynamic graph models of processes [5, 19].

DYNAMIC GRAPH MODELS

The fundamental feature of systems concerned is the natural decomposition (structure discretization) into sets of simple subsystems or structural states of S_i. The dynamics and the character of structural states interaction shall be defined by the pulse elements operating modes, modulation types, nonlinearities class, etc [4].

Graphs with time-varying elements (sets of vertices, edges or their weights) are called dynamic

$$G_t = < X_t\, (V_t, \Omega_t) >, \tag{1}$$

where X_t, V_t, Ω_t - accordingly vertex set, edges and edges coefficients.

$$X_t : t_* \to X;\ V_t : t_* \to V;\ \Omega_t : t_* \to \Omega;\ t_* = (t_1, t_2, ..., t_n)$$

– linearly ordered finite set of time instants.

There are different types of dynamic graphs. Structural state graphs describe a change in the system structure over time, while dynamic graph models of processes describe the direct dynamics of processes in individual structural states.

The formal models at the level of structural states are the dynamic graphs, the analytical description of which has the form as below

$$S_i = (X_i, R_i, \Omega_i), \tag{2}$$

where

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$S_i \in S = \{S_1, S_2, ..., S_n\}$;
$X_i \subseteq X = \{x_1, x_2, ..., x_m\}$;
$R_i \subseteq X_i \times X_i$;
$\Omega_i \subseteq \Omega = \{\omega_1, \omega_2, ..., \omega_k\}$;

S_i – the structural state of system, $\boldsymbol{X_i}$ – is the subset of system continuous part, match with the structural state S_i, x_j – the continuous system coordinates, R_i – binary relation in set X_i, ω_k – edge weight υ_k.

Down level models, meant for the processes description within the individual subsystems, shall be set by the graphs as

$$G_t = (X_t', X_t'', V_t), \tag{3}$$

where

$$X_t = X_t' \cup X_t''; \quad X_t' \cap X_t'' = \varnothing;$$
$$\forall x, y \in X_t \left[x, y \in V_t \Rightarrow x \in X_t' \ \&\ y \in X_t''\right];$$
$$X_t' = \left(x_1'(jT), x_2'(jT), ..., x_k'(jT)\right);$$
$$X_t'' = \left(x_1''(\overline{j+1}T), x_2''(\overline{j+1}T), ..., x_k''(\overline{j+1}T)\right);$$
$$\forall (x_i', x_j'') \in V_t \left[\Diamond(x_i', x_j'') = \nabla x_j'', x_i'\right];$$
$$i, j \in J = \{1, 2, ..., k\};$$

($\nabla x'', x' \Leftrightarrow$ «graph transmissions between the nodes $(\cdot,\cdot)$»; $\Diamond x_i', x_j'' \Leftrightarrow$ «edge weight $(\cdot,\cdot)$»; T – discretization period).

CONTROLLER DESIGN

Consider the discrete control system shown in Figure 1a. Block scheme of compensated system may be implemented with the aid of a sequential correction (Figure 1b). The digital controller transfer function $D(z)$ will be chosen on the basis of infinite degree of stability, i.e. the finite and minimum duration of the process.

The latter, besides, shall have monotonous (aperiodic) type. Digital controllers we present as amplifying elements with the varying gain k_j. Transient minimum time in the compensated system with the delay is equal to

$$t_{\min} = (l + \bar{\tau})T, \tag{4}$$

where l – is the degree of a differential equation of control object; $\bar{\tau}$ – is the relative time delay $\bar{\tau} = \tau / T$; T – is the pulse element switch period.

To obtain the system with the maximal response speed at piece-wise constant function inputs and zero initial conditions it is important, that, when $t < (l + \bar{\tau})T$ the output remains less than the input and the system error when $t \geq (l + \bar{\tau})T$ will be equal to zero. These restrains will be satisfied, if

$$y\left(\overline{l + \bar{\tau}\ T}\right) = f\left(\overline{l + \bar{\tau}\ T}\right), \tag{5}$$

$$\dot{y}\left(\overline{l + \bar{\tau}}T\right) = \ddot{y}\left(\overline{l + \bar{\tau}}T\right) = ... = y^{(l-1)}\left(\overline{l + \bar{\tau}}T\right) = 0\,. \tag{6}$$

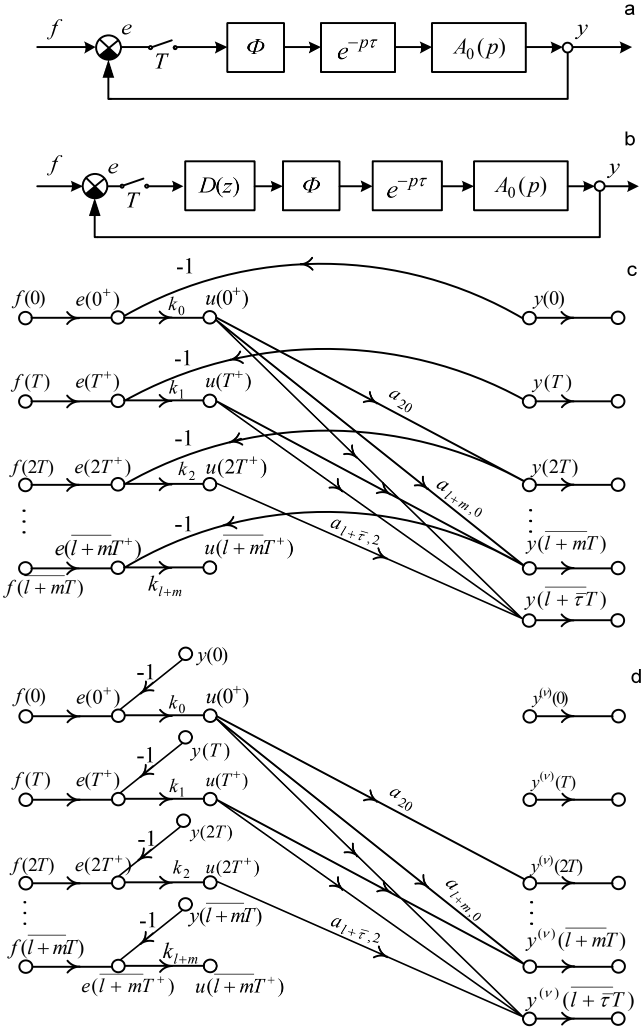

Fig. 1. Discrete control system with delay (a), sequential correction scheme (b), dynamic graph models of processes (c, d)

To determine the gain values k_j and digital controller transfer function $D(z)$ we construct output process dynamic graph models and its derivatives. In the Figure 1(c,d) the latter have been constructed for the l-order system with delay $\tau = 2T$. The union sub-graph $G_1(u_1, r_1)$ with the vertices

$$u_l = \left\{u(0^+), u(T^+), ..., u(\overline{l-1}T^+); y(\overline{l+\bar{\tau}}T)\right\} \tag{7}$$

and mappings

$$\begin{array}{l} r_l u(0^+) = \left\{y(\overline{l+\bar{\tau}}T)\right\} \\ r_l u(T^+) = \left\{y(\overline{l+\bar{\tau}}T)\right\} \\ \dots\dots\dots\dots\dots\dots\dots \\ r_l u(\overline{l-1}T^+) = \left\{y(\overline{l+\bar{\tau}}T)\right\} \end{array}, \tag{8}$$

sub-graph $G_2(u_2, r_2)$ with the vertices

$$u_2 = \left\{u(0^+), u(T^+), ..., u(\overline{l-1}T^+); \dot{y}(\overline{l+\bar{\tau}}T)\right\} \tag{9}$$

and mappings

$$\begin{array}{l} r_2 u(0^+) = \left\{\dot{y}(\overline{l+\bar{\tau}}T)\right\} \\ r_2 u(T^+) = \left\{\dot{y}(\overline{l+\bar{\tau}}T)\right\} \\ \dots\dots\dots\dots\dots\dots\dots \\ r_2 u(\overline{l-1}T^+) = \left\{\dot{y}(\overline{l+\bar{\tau}}T)\right\} \end{array} \tag{10}$$

and sub-graph $G_{l-1}(u_{l-1},r_{l-1})$ with the vertices

$$u_{l-1}=\left\{u(0^+),u(T^+),\dots,u(\overline{l-1T^+});y^{(l-1)}(\overline{l+\bar{\tau}T})\right\} \quad (11)$$

and mappings

$$\begin{aligned} &r_{l-1}u(0^+)=\left\{y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\\ &r_{l-1}u(T^+)=\left\{y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\\ &\dots\dots\\ &r_{l-1}u(\overline{l-1T^+})=\left\{y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\end{aligned} \quad (12)$$

allow to produce the fundamental system graph

$$G(u,r)=G_1(u_1,r_1)\cup G_2(u_2,r_2)\cup\dots\cup G_{l-1}(u_{l-1},r_{l-1}) \quad (13)$$

with the vertices

$$\begin{aligned} u=&\left\{u(0^+),u(T^+),\dots,u(\overline{l-1T^+});\right.\\ &\left.y(\overline{l+\bar{\tau}T}),\dot{y}(\overline{l+\bar{\tau}T}),\dots,y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\end{aligned} \quad (14)$$

and mappings

$$\begin{aligned} &ru(0^+)=\left\{y(\overline{l+\bar{\tau}T}),\dot{y}(\overline{l+\bar{\tau}T}),\dots,y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\\ &ru(T^+)=\left\{y(\overline{l+\bar{\tau}T}),\dot{y}(\overline{l+\bar{\tau}T}),\dots,y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\\ &\dots\dots\\ &ru(\overline{l-1T^+})=\left\{y(\overline{l+\bar{\tau}T}),\dot{y}(\overline{l+\bar{\tau}T}),\dots,y^{(l-1)}(\overline{l+\bar{\tau}T})\right\}\end{aligned} \quad . \quad (15)$$

In the above expressions, symbol r_1 denotes the mapping of each signal from the set of controls onto the output signal $y(\overline{l+\bar{\tau}T})$, r_2 is the mapping of each signal from the set of controls onto the derivative of output signal, and so on.

The graph constructed for the case l=3, τ=2T has shown in the Figure 2a. Using the essential graph corresponding transformations (nodes exception, invert, summation and so on), we will obtain the control actions graph (Figure 2b), based on which immediately we will determine the control actions values desired.

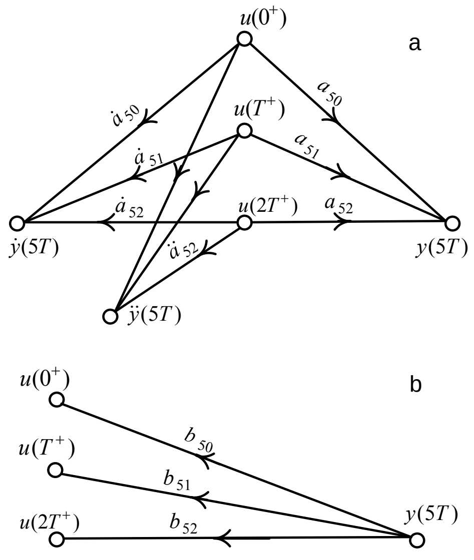

Fig. 2. An essential graph of system (a), graph of controls (b)

After, based on the dynamic graph model view (Figure 1c), we will determine output process values at the moments of pulse element quantification, i.e. value $y(0),y(T),y(2T),\dots,y(\overline{l+mT})$, where m is the integer.

Piecewise-constant gain factors shall be determined directly by the graph, or by formula

$$k_j=\frac{u(jT^+)}{f(jT)-y(jT)},\ (j\in J=\{0,1,2,\dots,l+m\}). \quad (16)$$

The required digital controller transfer function is found as the ratio of z-transforms of the control sequence and mismatch errors

$$D(z)=\frac{\sum_{j=0}^{l+m}k_j e(jT^+)z^{-j}}{\sum_{j=0}^{l+m}e(jT^+)z^{-j}}, \quad (17)$$

where $e(jT^+)=f(jT)-y(jT)$.

Obtained digital controller transfer function (17) ensures at the system output the aperiodic process of the finite and minimal duration, which differs from the process in the same system with no delay only by shifting by the delay time τ.

As the system output coordinate can be taken the water level deviation in the channel from the specified one. In this case could be applied the dynamic graph models modification – state variables graph.

Let the process initial state is characterized by vector $\vec{x}(0)\neq 0$, i.e. water level deviation from a given one. As the system equilibrium we assume the origin of coordinates the process state space. It is required to transfer the object from an initial state $\vec{x}(0)$ to zero state with the minimal numbers of discreteness steps when the impact delay terms of compensation are observed.

To find the control law we make use of the linear object properties, which consist in that, linear object of the l-th order with delay of $e^{-p\tau}$ may be transferred from any initial state of $\vec{x}(0)$ to the state of equilibrium in a time equal to $t_{min}=(l+\bar{\tau})T$, where

$$\vec{x}(\overline{l+\bar{\tau}T})=\vec{0}. \quad (18)$$

For the sake of simplicity assume, that, the relative delay time is equal to the whole number of $\bar{\tau}=m$, then (18) is transformed to the form

$$\vec{x}(\overline{l+mT})=\vec{0}. \quad (19)$$

Having constructed the state variables initial graph of concerned process and determined the transfers between the state variables, we expand the graph on the time interval $(0;\overline{l+mT})$ (Figure 3b).

We exclude the all intermediate vertices that match with the variables

$$\vec{x}(\overline{l+1T}),\ \vec{x}(\overline{l+2T}),\ \dots,\ \vec{x}(\overline{l+m-1T}).$$

Taking into account that the system state up to the time $t=mT$ is invariable, i.e. $\vec{x}(mT)=\vec{x}(\overline{m-1T})=\dots=\vec{x}(0)$, we obtain the variable state essential graph (Figure 4a), directly from which, considering (19), one can write down:

$$\left.\begin{array}{l} \varphi_1\left[\vec{x}(0),u(0),u(T),...,u(\overline{l-1T})\right]=0 \\ \varphi_2\left[\vec{x}(0),u(0),u(T),...,u(\overline{l-1T})\right]=0 \\ .... \\ \varphi_l\left[\vec{x}(0),u(0),u(T),...,u(\overline{l-1T})\right]=0 \end{array}\right\}. \quad (20)$$

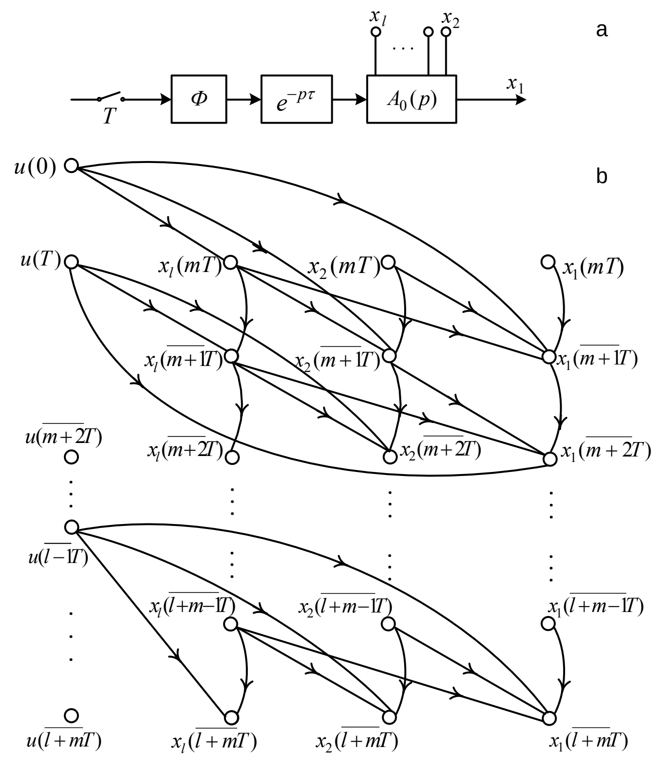

Fig. 3. Control system with delay (a), graph of the system deployed on the time interval (mT; l + mT) (b)

Transforming the graph (Figure 4a) so, as to obtain the transfers from the object initial state variables $x_1(0),x_2(0),...,x_l(0)$ to nodes $u(0),u(T),u(2T),...,u(\overline{l-1T})$, we get graph of controls shown in Figure 4b.

Based on the form of the latter let us determine the optimal control law

$$\left.\begin{array}{l} u(0)=\beta_{11}x_1(0)+\beta_{12}x_2(0)+...+\beta_{1l}x_l(0) \\ u(T)=\beta_{21}x_1(0)+\beta_{22}x_2(0)+...+\beta_{2l}x_l(0) \\ .... \\ u(\overline{l-1T})=\beta_{l1}x_1(0)+\beta_{l2}x_2(0)+...+\beta_{ll}x_l(0) \end{array}\right\}. \quad (21)$$

The resulting sequence of control actions depends on system initial state of $\vec{x}(0)$. Coefficients β_{lk} $(l \in L=\{1,2,...,l\}, k \in K=\{1,2,...,l\})$ are determined as a transformation result an essential graph into a graph of controls. Coefficients β_{lk} could also be obtained out of the algebraic equations system (20).

The control law in the current object states function may be obtained as follows. Invert the transmissions $x_i(mT)/x_j(\overline{l+mT})$ and perform the corresponding transformations of the significant graph, finally will be obtained the initial state graph (Figure 5a), from where one can write down:

$$\left.\begin{array}{l} r_{11}u(0)+r_{12}u(T)+...+r_{1l}u(\overline{l-1T})=x_1(0) \\ r_{21}u(0)+r_{22}u(T)+...+r_{2l}u(\overline{l-1T})=x_2(0) \\ .... \\ r_{l1}u(0)+r_{l2}u(T)+...+r_{ll}u(\overline{l-1T})=x_l(0) \end{array}\right\}. \quad (22)$$

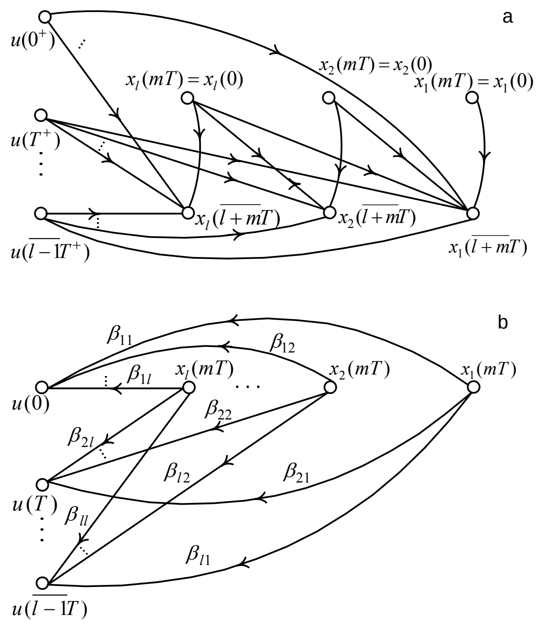

Fig. 4. An essential state variables graph (a), graph of controls (b)

Let us solve this linear algebraic equations system, or, what is the same, transform the graph (Figure 5a) so, as, to express the first control action $u(0)$ via $\vec{x}(0)$.

Be aware of the value $u(0)$ is enough, in order to transfer the object from the initial state $\vec{x}(mT) = \vec{x}(0)$ to the next state $\vec{x}(\overline{m+1T})$. Accepting the received state $\vec{x}(\overline{m+1T})$ as the new initial system state, let us repeat the task solution, i.e. according to the essential graph form for the interval $(\overline{m+1T};\ \overline{l+mT})$ we determine the value of the second control action, and so on.

With such approach the control law represents as the first formula out of the system (21), i.e.

$$u(0)=\beta_{11}x_1(0)+\beta_{12}x_2(0)+...+\beta_{1l}x_l(0). \quad (23)$$

The control law (23) could be practically realized in the form of a feedback operator (Figure 5b):

$$B=\beta_{11}+\beta_{12}p+...+\beta_{1l}p^l. \quad (24)$$

Based on mentioned above, we will formulate an algorithm for the synthesis of a control system with delay and non-zero initial conditions:

1. Construct the state variables graph of a system. Develop the graph on the interval $(0;\overline{l+mT})$, where m – relative delay equal to an integer; l – the order of the differential equation of a system.
2. Exclude the intermediate nodes $\vec{x}(\overline{l+1T}),\ \vec{x}(\overline{l+2T}),\ ...,\ \vec{x}(\overline{l+m-1T})$ and obtain an essential graph (Figure 4a), directly from which, considering (19), we write down (20).

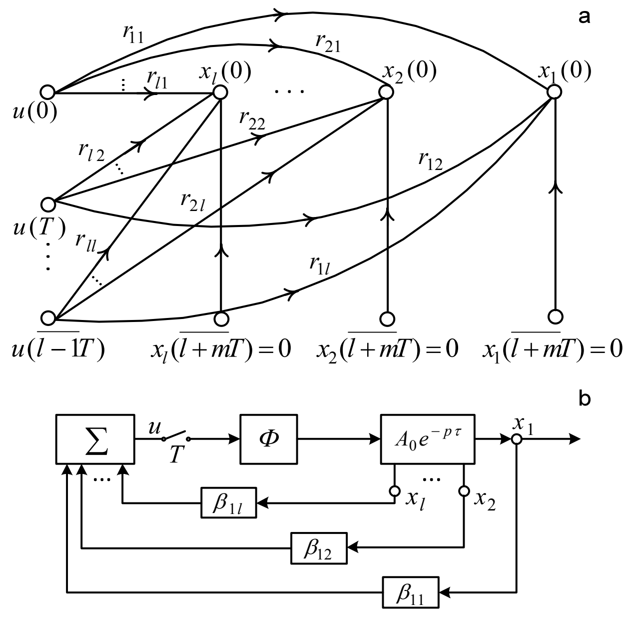

Fig. 5. State variables graph (a), synthesized scheme of the system (b)

3. Transforming the graph (Figure 4a), we get graph of controls (Figure 4b), by the form of which we write out the control law (21).
4. Invert the transmissions $x_i(mT)/x_j(\overline{l+mT})$ and perform the corresponding transformations of the significant graph, finally will be obtained the initial state graph (Figure 5a), from where write down (22).
5. To find the controls in the function of the current states of the object, we define the control actions $u(0)$ at the first step of the interrupt.
6. Accepting the received state $\vec{x}(\overline{m+lT})$ as the new initial system state, let us repeat the task solution, i.e. according to the essential graph form for the interval $(\overline{m+lT};\ \overline{l+mT})$ we determine the value of the second control action, and so on. So continue until the process goes into zero state.
7. The obtained control law we realize in the form of feedbacks by known methods.

In the case of a multivariable discrete system, the calculation sequence does not change. Multivariable control systems are characterized by *N*-control actions and *M*-output variables. The change of one control action causes a change in almost all output signals.

Construct the state variables graph of a multivariable system. Develop the graph on the interval $(0;\overline{n+mT})$, where *m* – relative delay equal to an integer; *n* – the nearest larger integer with respect to the quotient *l* / *N* (*l* is the order of the differential equation of system; *N* is the number of control actions or the "dimensionality" of system). Further, we act by analogy with the above algorithm.

PRACTICAL EXAMPLE

As an example, the formulated problem was solved for the Naiman hydro-technical Node control system (Figure 6a), the block diagram of which is presented as the discrete system (Figure 6b). Pulse elements operate in synchronous-in-phase mode with a period T = 900sec. The transfer function of a multivariable object is given in the form

$$\vec{A}_0(p)=\begin{bmatrix}\dfrac{0.0484e^{-1.5pT}}{p(858.6p+1)} & \dfrac{0.872e^{-1.5pT}}{p(1554.6p+1)}\\ \dfrac{0.0856e^{-1.5pT}}{p(121.3p+1)^2} & \dfrac{0.0305e^{-1.5pT}}{p(616.5p+1)}\\ \dfrac{0.0912e^{-1.5pT}}{p(768.8p+1)} & \dfrac{0.664e^{-1.5pT}}{p(831.5p+1)}\end{bmatrix}$$

The same delays in the separate transmission canals are due to the corresponding arrangement of measuring wells in each canal.

It was required to find the controllers transfer functions, providing compensation of the delay influence, aperiodic nature of transient processes at the system outputs, and their finite and minimal duration.

Applying the results of section 4 and relations (16), (17), we found the gain factors and impulse transfer functions of digital controllers

$$k_0^1=\frac{u^1(0^+)}{f^1(0)-y^1(0)}=0.0348;\ k_0^2=-0.0246;$$

$$k_1^1=-0.0254;\ k_1^2=0.0288.$$

$$D^1(z)=\frac{0.0348-0.0254z^{-1}}{1+z^{-1}+0.965z^{-2}+0.511z^{-3}};$$

$$D^2(z)=\frac{-0.0246+0.0288z^{-1}}{1+z^{-1}+0.817z^{-2}+0.0684z^{-3}}.$$

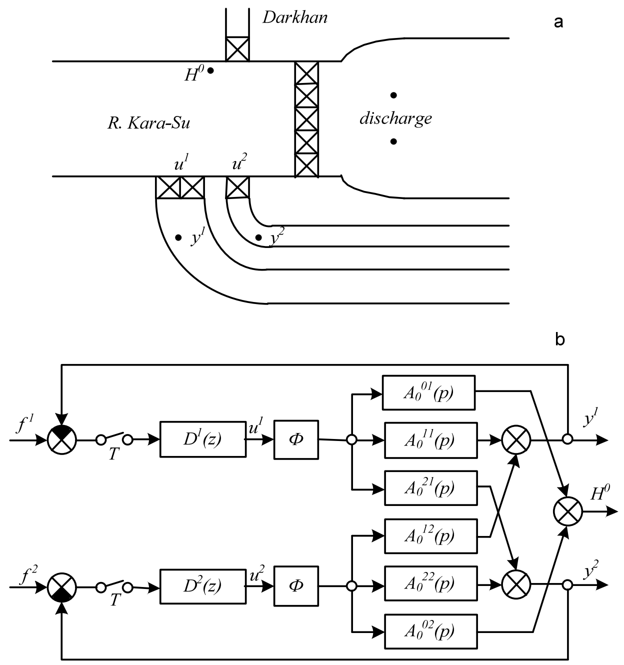

Fig. 6. Naiman hydro-technical Node (a), schematic representation of the Node control system (b)

The numerical values of the output functions y^1, y^2 are given in the table. Figure 7 shows the process curves obtained at the outputs of the system.

nT	*0.5T*	*T*	*1.5T*	*2T*	*2.5T*	*3T*	*3.5T*	*4T*
y^1	*0*	*0*	*0*	*0.0348*	*0.0615*	*0.489*	*1*	*1*
y^2	*0*	*0*	*0*	*0.182*	*0.612*	*0.932*	*1*	*1*
H^0	*0*	*0*	*0*	*-0.00038*	*-0.00051*	*0.0004*	*0.00087*	*0.00046*

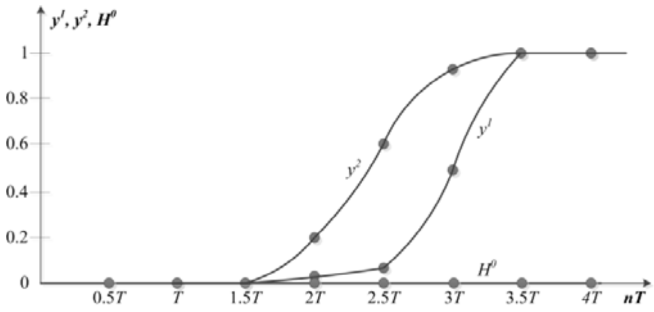

Fig. 7. Output processes curves

CONCLUSIONS

Application of dynamic graphs method allows to easily coping with such control systems simulation complexity factor like delay. The presence of inertia and delay only simplifies the graph structure, since it is turned into an exception of the corresponding edges. The dynamic graph models allow to calculate system dynamics over the all coordinates of interest, to synthesize control laws according to the chosen optimality criteria (response speed, mismatch errors minimization, efficiency and others).To the purpose of illustration of approach suggested have been considered the Naiman hydro-technical Node control system.

The approach can be applied to solve the problem of synthesis of discrete systems with variable frequency of interruption, with multiple synchronized and no synchronized interrupt frequencies, systems with pulse modulation in duration, frequency, systems with finite pulse duration, etc.

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